Från kapitel 13 - exempel med normalfördelade data, standardiserade normalfördelningen, Z-värde och sannolikhet med R-kod resp. sannolikhetstabell

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GUIDED PRACTICE

SAT scores follow a nearly normal distribution with a mean of 1500 points and a standard deviation of 300 points. ACT scores also follow a nearly normal distribution with mean of 21 points and a standard deviation of 5 points. Suppose Nel scored 1800 points on their SAT and Sian scored 24 points on their ACT. Who performed better?[3]

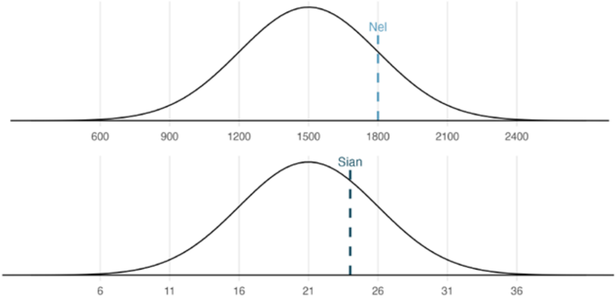

Figure 13.5: Nel's and Sian's scores shown with the distributions of SAT and ACT scores

$$Z = \frac{x - \mu}{\sigma}$$

$$Z_{Nel} = \frac{x_{Nel} - \mu_{SAT}}{\sigma_{SAT}} = \frac{1800 - 1500}{300} = 1$$

$$Z_{Sian} = \frac{x_{Sian} - \mu_{ACT}}{\sigma_{ACT}} = \frac{24 - 21}{5} = 0.6$$

GUIDED PRACTICE

Head lengths of brushtail possums follow a nearly normal distribution with mean 92.6 mm and standard deviation 3.6 mm. Compute the Z scores for possums with head lengths of 95.4 mm and 85.8 mm.[5]

For $x_1 = 95.4$ mm: $Z_1 = \frac{x_1 - \mu}{\sigma} = \frac{95.4 - 92.6}{3.6} = 0.78$.

For $x_2 = 85.8$ mm: $Z_2 = \frac{85.8 - 92.6}{3.6} = -1.89$.

13.2.3 Normal probability calculations

Nel from the SAT Guided Practice earned a score of 1800 on their SAT with a corresponding $Z = 1$. They would like to know what percentile they fall in among all SAT test-takers.

Nel's **percentile** is the percentage of people who earned a lower SAT score than Nel. We shade the area representing those individuals in Figure 13.6. The total area under the normal curve is always equal to 1, and the proportion of people who scored below Nel on the SAT is equal to the *area* shaded in Figure 13.6: 0.8413. In other words, Nel is in the 84^{th} percentile of SAT takers.

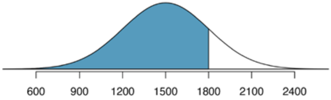

Figure 13.6: The normal model for SAT scores, shading the area of those individuals who scored below Nel.

Hur får vi 0.84?

1. I detta fall (Z=1) kan vi använda 68/95/99.7-regeln (area till vänster om 1800 är 0.5 + 0.68/2 = 0.5 + 0.34 = 0.84

2. R: pnorm

```
> pnorm(1800, mean=1500, sd=300)
[1] 0.8413447
```

3. Standardiserade normalfördelningstabellen (finns på Athena)

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214

Ta fram sannolikhet under ett visst Z-värde

Med R:

Normal probabilities are most commonly found using statistical software which we will show here using R. We use the software to identify the percentile corresponding to any particular Z score. For instance, the percentile of $Z = 0.43$ is 0.6664, or the 66.64^{th} percentile. The `pnorm()` function is available in default R and will provide the percentile associated with any cutoff on a normal curve. The `normTail()` function is available in the **openintro** R package and will draw the associated normal distribution curve.

```
pnorm(0.43, mean = 0, sd = 1)
```

```
[1] 0.666
```

```
openintro::normTail(m = 0, s = 1, L = 0.43)
```

Med tabell:

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0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793

Ta fram det Z-värde som har en viss sannolikhet till vänster i fördelningen

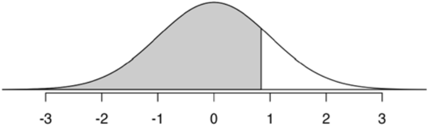
Med R:

We can also find the Z score associated with a percentile. For example, to identify Z for the 80^{th} percentile, we use `qnorm()` which identifies the **quantile** for a given percentage. The quantile represents the cutoff value. (To remember the function `qnorm()` as providing a cutozff, notice that both `qnorm()` and "cutoff" start with the sound "kuh". To remember the `pnorm()` function as providing a probability from a given cutoff, notice that both `pnorm()` and probability start with the sound "puh".) We determine the Z score for the 80^{th} percentile using `qnorm()`: 0.84.

```
qnorm(0.80, mean = 0, sd = 1)
```

```
[1] 0.842
```

```
openintro::normTail(m = 0, s = 1, L = 0.842)
```



Med tabell:

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
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0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
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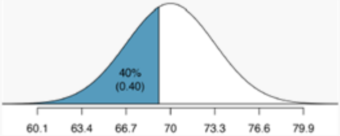
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327

Based on a sample of 100 men,[10] the heights of adults who identify as male, between the ages 20 and 62 in the US is nearly normal with mean 70.0" and standard deviation 3.3".

Yousef's height is at the 40^{th} percentile. How tall are they?
As always, first draw the picture.



In this case, the lower tail probability is known (0.40), which can be shaded on the diagram. We want to find the observation that corresponds to the known probability of 0.4. We can find the observation in two different ways: using the height curve seen above or using the Z score associated with the standard normal curve centered at zero with a standard deviation of one.

If you have access to software (like R, code seen below) that allows you to specify the mean and standard deviation of the normal curve, you can calculate the observed value on the curve (i.e., Yousef's height) directly.

```
[1] 69.2
```

```
> qnorm(0.4, mean=70, sd=3.3)
[1] 69.16395
```

Yousef is 69.2 inches tall. That is, Yousef is about 5'9" (this is notation for 5-feet, 9-inches).

Without access to flexible software, you will need the information given by a standard normal curve (a normal curve centered at zero with a standard deviation of one). First, determine the Z score associated with the 40^{th} percentile.

Because the percentile is below 50%, we know Z will be negative. Statistical software provides the Z value to be -0.25.

R:
```
qnorm(0.4, mean = 0, sd = 1)
[1] -0.253
```

Tabell: Se nästa sida

Knowing $Z_{Yousef} = -0.25$ and the population parameters $\mu = 70$ and $\sigma = 3.3$ inches, the Z score formula can be set up to determine Yousef's unknown height, labeled x_{Yousef}:

$$-0.253 = Z_{Yousef} = \frac{x_{Yousef} - \mu}{\sigma} = \frac{x_{Yousef} - 70}{3.3}$$

Solving for x_{Yousef} yields the height 69.2 inches. Again, Yousef is about 5'9".

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.9	.00005	.00005	.00004	.00004	.00004	.00004	.00004	.00004	.00003	.00003
-3.8	.00007	.00007	.00007	.00006	.00006	.00006	.00006	.00005	.00005	.00005
-3.7	.00011	.00010	.00010	.00010	.00009	.00009	.00008	.00008	.00008	.00008
-3.6	.00016	.00015	.00015	.00014	.00014	.00013	.00013	.00012	.00012	.00011
-3.5	.00023	.00022	.00022	.00021	.00020	.00019	.00019	.00018	.00017	.00017
-3.4	.00034	.00032	.00031	.00030	.00029	.00028	.00027	.00026	.00025	.00024
-3.3	.00048	.00047	.00045	.00043	.00042	.00040	.00039	.00038	.00036	.00035
-3.2	.00069	.00066	.00064	.00062	.00060	.00058	.00056	.00054	.00052	.00050
-3.1	.00097	.00094	.00090	.00087	.00084	.00082	.00079	.00076	.00074	.00071
-3.0	.00135	.00131	.00126	.00122	.00118	.00114	.00111	.00107	.00104	.00100
-2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
-2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
-2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264

Tabell fortsätter..

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-1.0	.15866	.15625	.15386	.15151	.14917	.14686	.14457	.14231	.14007	.13786
-0.9	.18406	.18141	.17879	.17619	.17361	.17106	.16853	.16602	.16354	.16109
-0.8	.21186	.20897	.20611	.20327	.20045	.19766	.19489	.19215	.18943	.18673
-0.7	.24196	.23885	.23576	.23270	.22965	.22663	.22363	.22065	.21770	.21476
-0.6	.27425	.27093	.26763	.26435	.26109	.25785	.25463	.25143	.24825	.24510
-0.5	.30854	.30503	.30153	.29806	.29460	.29116	.28774	.28434	.28096	.27760
-0.4	.34458	.34090	.33724	.33360	.32997	.32636	.32276	.31918	.31561	.31207
-0.3	.38209	.37828	.37448	.37070	.36693	.36317	.35942	.35569	.35197	.34827
-0.2	.42074	.41683	.41294	.40905	.40517	.40129	.39743	.39358	.38974	.38591
-0.1	.46017	.45620	.45224	.44828	.44433	.44038	.43644	.43251	.42858	.42465
-0.0	.50000	.49601	.49202	.48803	.48405	.48006	.47608	.47210	.46812	.46414

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